

# Resource Pack for Secondary Schools

## Answers

Answers to questions are listed in the order they appear from top to bottom on the Resource Sheet.

### SR1–SR3: Scottish Mathematical Challenge questions

The answers to these questions are not in the public domain at the time of going to press. They will be available on the SMC website shortly after publication: <http://www-maths.mcs.st-and.ac.uk/~smc/>

### SR4–SR5: Mathématiques sans frontières

The answers are available on the Mathématiques sans frontières website: <http://www.hw.ac.uk/msf/>

### SR6: UKMT Questions (S1–S2)

- E    C    D    D    E    E

### SR7: UKMT Questions (up to S5)

- B    B    B    D    B    A    D

### SR8: UKMT Questions (up to S6)

- C    D    B    D    E    A    A    E

### SR9: Fabulous Fibonacci

- (a) 1   (b) 2   (c) 3   (d) 13   (e) Add the results of the previous two numbers
- (a) 55   (b) 0   (c) 1   (d) -1   (e) 5
- (f)  $F_n = \begin{cases} F_n & \text{if } n \text{ is odd} \\ -F_n & \text{if } n \text{ is even} \end{cases}$
- $F_n^2 = F_{n-1} \times F_{n+1} \pm 1$

### SR10: Contest Questions

- 28 palindromes (excluding 0)    16 years    11  
 ratio 1:2    20 cans



- A=3, B=1, C=7, D=5, E=9
- $98 - 76 + 54 + 3 + 21 = 100$

**SR14: Quiz Round**

*10 starters for 20*

- 30
- $\frac{9}{20}$
- 0.225
- 50,000
- 45m
- $\frac{13}{20}$
- $x = 10$
- $8x^3$
- 13cm
- $\frac{1}{6}$

*Time and time again*

- 7.20am

**SR15: Quiz Round**

*Magic Square*

|    |    |    |    |
|----|----|----|----|
| 1  | 14 | 7  | 12 |
| 15 | 4  | 9  | 6  |
| 10 | 5  | 16 | 3  |
| 8  | 11 | 2  | 13 |

*The good, the bad and the tricky sum*

$$\begin{array}{r}
 7 \ 2 \ 0 \\
 + \ 7 \ 2 \ 0 \\
 \hline
 1 \ 4 \ 4 \ 0
 \end{array}$$

$$\begin{array}{r}
 8 \ 3 \ 0 \\
 + \ 8 \ 3 \ 0 \\
 \hline
 1 \ 6 \ 6 \ 0
 \end{array}$$

$$\begin{array}{r}
 9 \ 4 \ 0 \\
 + \ 9 \ 4 \ 0 \\
 \hline
 1 \ 8 \ 8 \ 0
 \end{array}$$

*Drives you mad*

1 hour 40 minutes

### SR16: Quiz Round

*What comes next?*

|                |               |
|----------------|---------------|
| 8, 13          | Fibonacci     |
| 31, 41         | Prime time!   |
| 122, 365       | Times and add |
| S, S           | en Francais   |
| 131221, 132231 | WYSIWYG       |

*Crossnumber*

#### Across

1 784 ( $28^2$ )

3 975

#### Down

1 729 ( $9^3$ )

2 465

*On the Carpet*

maximum  $200.5 \times 150.5 = 30,175.25\text{cm}^2$

minimum  $199.5 \times 149.5 = 29,825.25\text{cm}^2$

difference  $350\text{cm}^2$

### SR17: Quiz Round

*Birthday treat*

4x4 5    3x3 12    2x2 21    1x1 32

total 70

*Easter eggstravaganza*

10

*Roman all over the place*

300 (R, K, S, M, R, C, U, R)

**SR18: Quiz Marathon**

- 1) 147
- 2) 9
- 3) 76
- 4) 360
- 5) 25
- 6) 1914
- 7) 40
- 8) 7
- 9) 9
- 10) 8
- 11) 192
- 12) 12
- 13) 10
- 14) 144
- 15) 12
- 16) 20
- 17) 13
- 18) 8
- 19) 200
- 20) 12
- 21) 1024
- 22) 2001
- 23) 366
- 24) 12
- 25) 20
- 26) 240
- 27) 101
- 28) 1000
- 29) 12
- 30) 0131
- 31) 13
- 32) 4
- 33) 6
- 34) 2000
- 35) 25
- 36) 180
- 37) 700
- 38) 5
- 39) 22
- 40) 4
- 41) 7
- 42) 14
- 43) 5
- 44) 1471
- 45) 8
- 46) 50
- 47) 21
- 48) 10,000
- 49) 21
- 50) 40

**SR19: Number Challenge**

- 1) t
- 2) 12
- 3) 4
- 4) 26 miles and 385 yards  
(accept 26 miles)
- 5) 2 rabbits (5 pigeons)
- 6) 3
- 7) 6, 8, 8, 20
- 8) a decimal point
- 9) 4
- 10) Thursday
- 11) 11
- 12) 9
- 13) 2999
- 14) 23 years old
- 15) 300
- 16) 144
- 17) 20:02:2002
- 18) 10
- 19) 24 minutes
- 20) 100,000 weeks or  
1923 years plus 4  
weeks
- 21) 12
- 22) Thursday
- 23) 20
- 24) 5 seconds
- 25) 14
- 26) 14
- 27) 10
- 28) 280
- 29) 12
- 30) 1, 3, 7, 9
- 31) 2464
- 32) 24

**SR21: Mathematical Bingo  
Questions**

- 1) 7
- 2) 24
- 3) 9
- 4) 12
- 5) 21
- 6) 20
- 7) 23
- 8) 13
- 9) 25
- 10) 18
- 11) 17
- 12) 2
- 13) 11
- 14) 10
- 15) 5
- 16) 19
- 17) 8
- 19) 3
- 20) 15
- 21) 14
- 22) 6
- 23) 4
- 24) 22
- (25) 1

SR23: Funny Money

- Using 5sc and 8sc notes, these amounts can be made: 20sc, 25sc, 13sc, 32sc, 42sc, 51sc, 132sc, 937sc, 1021sc, and these amounts can't be made: 6sc, 19sc.
- Using combinations of 5sc and 8sc notes the amounts, up to 50sc, which can be made are: 5sc, 8sc, 10sc, 13sc, 15sc, 16sc, 18sc, 20sc, 21sc, 23sc, 24sc, 25sc, 26sc, 28sc, and all amounts above this
- Using combinations of 5sc and 9sc notes the amounts up to 50sc are: 5sc, 9sc, 10sc, 14sc, 15sc, 18sc, 19sc, 20sc, 23sc, 24sc, 25sc, 27sc, 28sc, 29sc, 30sc, 32sc and all amounts above.

| $a = 5$ and $b = 2$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |
|---------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| Number              | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| O.K.?               |   | ✓ |   | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  |
| Useless?            | ✓ |   | ✓ |   |   |   |   |   |   |    |    |    |    |    |    |

| $a = 5$ and $b = 3$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |
|---------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| Number              | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| O.K.?               |   |   | ✓ |   | ✓ | ✓ |   | ✓ | ✓ | ✓  | ✓  | ✓  | ✓  | ✓  | ✓  |
| Useless?            | ✓ | ✓ |   | ✓ |   |   | ✓ |   |   |    |    |    |    |    |    |

| $a = 5$ and $b = 4$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |
|---------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| Number              | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| O.K.?               |   |   |   | ✓ | ✓ |   |   | ✓ | ✓ | ✓  |    | ✓  | ✓  | ✓  | ✓  |
| Useless?            | ✓ | ✓ | ✓ |   |   | ✓ | ✓ |   |   |    | ✓  |    |    |    |    |

| $a = 5$ and $b = 5$ |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |
|---------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| Number              | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| O.K.?               |   |   |   |   | ✓ |   |   |   |   | ✓  |    |    |    |    | ✓  |
| Useless?            | ✓ | ✓ | ✓ | ✓ |   | ✓ | ✓ | ✓ | ✓ |    | ✓  | ✓  | ✓  | ✓  |    |

- The results for  $a = 5$  and  $b = 5$  are very easy to understand. The only possible **O.K.** numbers are multiples of 5

| $a = 5$ and $b = 6$ |    |    |    |    |    |    |    |    |    |    |    |    |
|---------------------|----|----|----|----|----|----|----|----|----|----|----|----|
| Number              | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
| O.K.?               |    |    |    |    | ✓  | ✓  |    |    |    | ✓  | ✓  | ✓  |
| Useless?            | ✓  | ✓  | ✓  | ✓  |    |    | ✓  | ✓  | ✓  |    |    |    |
| Number              | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| O.K.?               |    |    | ✓  | ✓  | ✓  | ✓  |    | ✓  | ✓  | ✓  | ✓  | ✓  |
| Useless?            | ✓  | ✓  |    |    |    |    | ✓  |    |    |    |    |    |

5. There are many patterns which occur in the tables, like clusters of **O.K.** and **Useless** numbers.
6. If the student is at all unsure about what will happen for  $a = 5$  and  $b = 7$ , then he should make a table similar to the ones which have gone before.
7. The most easily understood explanation of the fact that there seems a point beyond which there are no more **Useless** numbers relies on the clusters of **O.K.** numbers. Once there are five in a row you can easily produce the next number by taking the first of the cluster of five **O.K.** numbers and adding 5 to it.
8. The final **Useless** number for  $a = 5$  and  $b = 8$  is 27.
9. It is a good idea to collect the results in a chart before looking for the connection.

|                            |   |   |    |   |    |    |    |    |    |
|----------------------------|---|---|----|---|----|----|----|----|----|
| $a$                        | 5 | 5 | 5  | 5 | 5  | 5  | 5  | 5  | 5  |
| $b$                        | 2 | 3 | 4  | 5 | 6  | 7  | 8  | 9  | 10 |
| last <b>Useless</b> number | 3 | 7 | 11 | ? | 19 | 23 | 27 | 31 | ?  |

The connection between  $a$ ,  $b$  and the last **Useless** number is  $a \times b - (a + b)$ . So for  $a = 5$  and  $b = 8$  it is  $5 \times 8 - (5 + 8) = 40 - 13 = 27$

10. It is worth suggesting an investigation where the two available notes have a common divisor. For example  $a = 6$  and  $b = 4$  will provide a completely different type of result as both  $a$  and  $b$  are multiples of 2.

### SR31: Countdown

- 1  $7 \div 7 = 1, 100 + 3 + 6 - 1 = 108.$
- 2  $2 + 7 = 9, 9 \times 100 = 900 + 5 + 1 = 906.$
- 3  $3 \times 7 = 21, 21 + 8 = 29 \times 25 = 725 - 10 = 715.$
- 4  $75 + 7 = 82 \times 10 = 820 + (9 \div 9) + 5 = 82$   
or  $75 + 9 = 84 \times 10 = 840 - (9 + 5) = 826.$
- 5  $(5 - 2) = 3 \times 4 \times 8 = 96, 96 \times 10 = 960 - 9 = 951.$
- 6  $6 + 4 = 10 \times 75 = 750 + 3 = 753.$
- 7  $75 - 2 - 3 = 70, 70 \times 8 = 560 + 7 - 1 = 566$   
or  $75 \times 7 = 525. (2 + 3) \times 8 = 40. 525 + 1 + 40 = 566.$   
or  $75 \times 8 = 600. (3 + 2) \times 7 = 35. 600 - 35 + 1 = 566.$

8.  $(75 + 3) \times (10 + 2) = 936 + 6 + 3 = 945$   
or  $10 + 3 = 13$ .  $13 \times 75 = 975$ .  $(3 + 2) \times 6 = 30$ .  $975 - 30 = 945$ .
9.  $100 + 75 + 25 + (9 - 8) = 201$
10.  $(5 + 1 + 6) \times 50 + 9 = 609$
11.  $75 \times 6 = 450$ .  $450 - 3 = 447$ .
12.  $(75 + 6) \times 3 = 243$ .  $243 + 5 = 248$ .
13.  $3 \times 9 = 27$ .  $27 \times 25 = 675$ .  $8 \div 2 = 4$ .  $675 + 4 = 679$ .
14.  $50 + 5 = 55$ .  $55 \times 6 = 330$ .  
or  $6 \times 50 = 300$ .  $3 \times 2 \times 5 = 30$ .  $300 + 30 = 330$ .
15.  $10 \times 75 = 750$ .  $10 - 6 = 4$ .  $750 - 50 + 4 = 704$ .  
or  $75 - 5 = 70$ .  $70 \times 10 = 700$ .  $10 - 6 = 4$ .  $700 + 4 = 704$ .
16.  $(3 \times 9) + 5 + 25 = 57$ .  $57 \times 10 = 570$ .  $570 - 9 = 561$ .
17.  $9 \times 8 \times 10 = 720$ .  $3 \times 5 = 15$ .  $720 + 50 + 15 = 775$ .  
or  $50 \times 3 = 150$ .  $150 + 8 - (10 - 9) = 157$ .  $157 \times 5 = 785$ .
18.  $7 \times 2 = 14$ .  $14 \times 50 = 700$ .  $9 - 5 = 4$ .  $700 - 4 = 696$ .  
or  $(9 + 5) \times 50 = 700$ .  $8 \div 2 = 4$ .  $700 - 4 = 696$ .
19.  $(2 + 3) \times 100 = 500$ .  $10 - 2 = 8$ .  $500 + 8 = 508$ .
20.  $(75 - 3) \times 10 = 720$ .  $8 - 7 = 1$ .  $7 - 1 = 6$ .  $720 - 6 = 714$ .
21.  $4 \times 75 = 300$ .  $9 \times 3 = 27$ .  $300 + 27 = 327$ .  
or  $75 + 8 = 83$ .  $83 \times 4 = 332$ .  $332 - 3 - 2 = 327$ .
22.  $10 \times 25 = 250$ .  $(250 + 8) \times 3 = 774$ .  $774 - 1 = 773$ .
23.  $75 \times 3 = 225$ .
24.  $75 \times 8 = 600$ .  $5 \times 5 + 10 = 35$ .  $600 - 35 = 565$ .  
or  $(75 - 5) \times 8 = 560$ .  $560 + 5 = 565$ .
25.  $(6 \times 5 + 4) \times 25 = 850$ .  $850 + 9 + 3 = 862$ .
26.  $(7 \times 50) - (9 - 8) = 349$ .

27.  $(2 \times 4 + 10) \times 25 = 450$ .  $450 - (5 - 1) = 446$ .
28.  $(8 + 9) \times 50 = 850$ .  $6 \times 5 + 3 = 33$ .  $850 - 33 = 817$ .
29.  $(5 \times 25) - (9 - 6) = 122$ .
30.  $(3 + 5) \times 9 = 72$ .  $6 \div 3 = 2$ .  $100 + 72 - 2 = 170$ .
31.  $(9 + 1) \times 75 = 750$ .  $100 \div 25 = 4$ .  $750 + 4 = 754$ .
32.  $5 \times 75 - (10 + 7 + 6) = 352$ .
33.  $100 + (7 - 6) + 2 = 103$ .
34.  $75 \times 9 = 675$ .  $100 \div 50 = 2$ .  $675 + (25 - 6 - 2) = 692$ .
35.  $75 \times 4 = 300$ .  $9 \times 4 = 36$ .  $300 - 36 = 264$ .
- or  $(75 - 9) \times 4 = 264$ .
36.  $2 \times 5 \times 75 = 750$ .  $4 \times 6 = 24$ .  $750 - 24 = 726$ .
37.  $(10 + 10) \times 25 = 500$ .  $500 + 8 - 2 = 506$ .
38.  $100 + 7 = 107$ .  $(4 + 4) \times 107 = 856$ .  $856 + 6 + 3 = 865$ .
39.  $9 \times 50 = 450$ .  $75 \div 25 = 3$ .  $450 - 100 - 3 - 2 = 345$ .
40.  $75 \times 5 = 375$ .  $3 \times 7 + 8 = 29$ .  $375 - 29 = 346$ .
41.  $5 \times 100 + 8 = 508$ .
42.  $6 \times 100 - 5 = 595$ .
43.  $8 \times 50 + 10 = 410$ .  $410 \times 2 = 820$ .  $820 + 6 + 5 = 831$ .
44.  $6 \times 9 \times 10 = 540$ .  $540 + 7 + 1 = 548$ .
45.  $(50 + 2) \times 8 = 416$ .  $100 \div 25 = 4$ .  $416 - 4 = 412$ .
46.  $4 \times 100 + 75 + 9 = 484$ .
47.  $(75 + 7) \times 6 = 492$ .  $492 + 9 = 501$ .
- or  $(75 \times 7) = 525$ .  $(3 + 1) \times 6 = 24$ .  $525 - 24 = 1$ .
48.  $(75 - 3) \times 10 = 720$ .  $100 \div 50 = 2$ .  $720 - 2 = 718$ .
49.  $(3 \times 10) - 1 = 29$ .  $29 + 75 = 104$ .
50.  $(6 \times 4) - 1 = 23$ .  $23 \times 25 = 575$ .

51.  $(7 - 3) \times 50 = 200.$   $10 \div 2 = 5.$   $200 + 5 = 205.$
52.  $3 \times 100 + 10 = 310.$
53.  $(25 - 5) \times 50 = 1000.$   $1000 - (10 + 1) = 989.$
54.  $(9 + 1) \times 50 = 500.$   $500 - (5 + 8) = 487.$
55.  $(10 - 2) \times 50 = 400.$   $75 \div 25 = 3.$   $400 + 6 \div 3 = 402.$   
or  $75 \times 6 = 450.$   $450 - 50 = 400.$   $400 + 2 = 402$   
or  $(10 + 6) \times 25 = 400.$   $400 + 2 = 402.$
56.  $(8 \times 25) - 10 = 190.$
57.  $(7 + 2) \times 100 = 900.$   $4 \times 7 + 50 = 78.$   $900 - 78 = 822.$
58.  $9 \times 10 \times 10 + 2 = 902.$   
or  $9 \times 50 = 450.$   $(450 + 1) \times 2 = 902.$
59.  $(50 + 7) \times (4 + 7) = 627.$
60.  $7 \times 100 = 700.$   $(7 - 1) \times 6 = 36.$   $700 + 36 = 736.$

### SR32: Targets

Examples of solutions:

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| 1. $6 \times 5 + 3 - 2 = 31$         | 2. $7 \times 4 + 2 - 5 = 25$         |
| 3. $(8 - 3) \times 7 + 6 = 41$       | 4. $2 \times 5 + 9 + 4 = 23$         |
| 5. $(6 + 8) \times 3 + 9 = 51$       | 6. $(7 \times 4 - 3) \times 2 = 50$  |
| 7. $9 \times 6 - 4 + 7 = 57$         | 8. $(8 \times 5 - 2) \times 3 = 114$ |
| 9. $(9 - 3) \times 2 + 7 = 19$       | 10. $6 \times 8 - 7 \times 4 = 20$   |
| 11. $(5 \times 4 + 8) \times 3 = 84$ | 12. $7 \times 8 + 2 - 3 = 55$        |

### SR34: Mathematics in the real world

- 8.4 metres
- 5.25%
- 3128.9 cubic yards; roads department needs 5632 tons; requires 226 trucks

- Child ingests 3.6 grams, in danger of at least renal failure
- 95.454545...mg or approximately 95.5 mg.
- Approximately 12.9 seconds
- Approximately 2.0 years (rounded); 0.0000148 inches worn off per stop; 0.0000167 inches worn off per mile
- 4L for £53.57, 1.3p per millilitre
- Answers will vary; for example, if one considers an average life span to be 75 years and an average night's sleep to be 8 hours, then one would sleep 25 years.
- 25.2 cubic metres or 889.9 cubic feet; more than adequate with 105.3 cubic metres or 3718.1 cubic feet to spare. (Warehouse holds 4608 cubic feet or 130.5 cubic metres of space.)
- 176,400 cubic feet (by one method); 9,800 boxes if stacked on the largest face for stability: other answers are possible if the boxes are stacked on smaller faces, or if racks or shelves are used (many answers, good for class discussions)